Learning Objective

I will be able to define the domain and range of a function

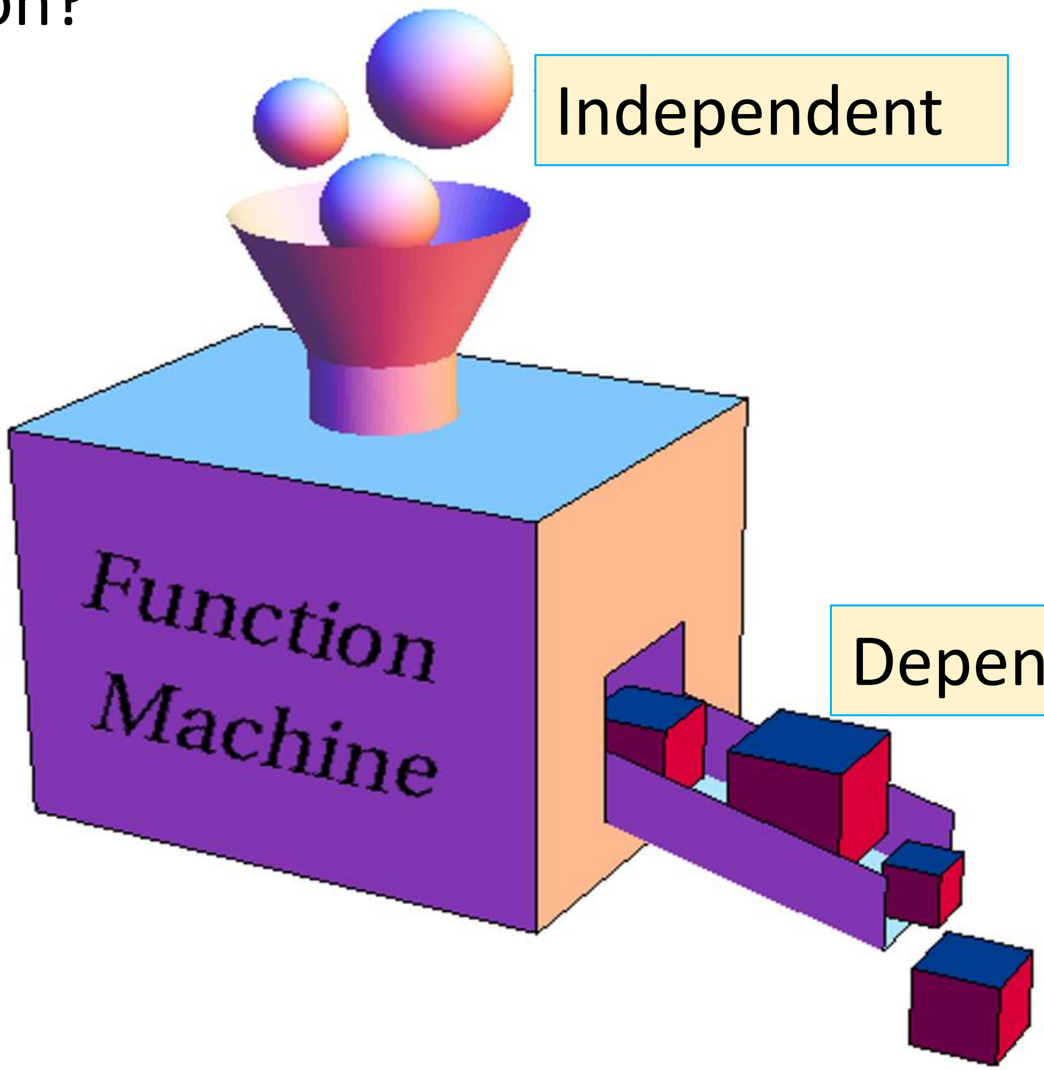
Success Criteria

At the end of the lesson, I will be able to:

- **define the domain and range for the line segment of a linear function.**

Concept Development

What is a function?



Independent

Dependent

Functions

- A function is a rule which maps **each x value** to just **one y value** for a defined set of input values.

OR

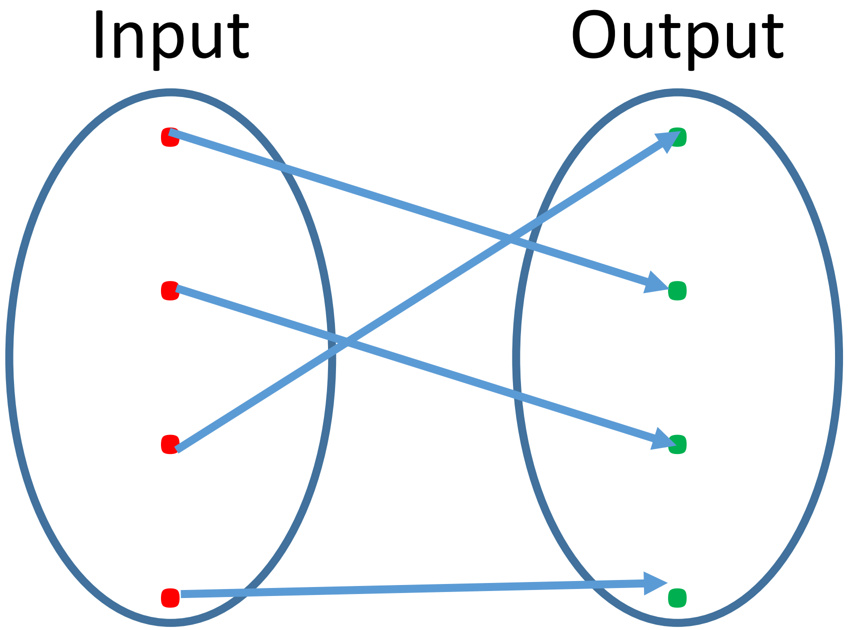
- A function is a relation in which **NO** two different ordered pairs have the **same x – coordinates**.

$$y = 6x - 2$$

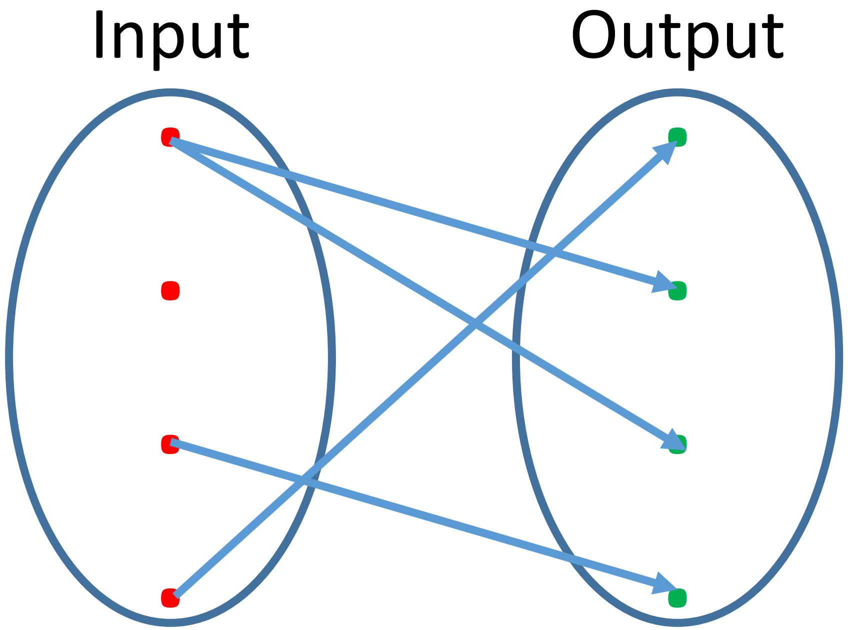
Write this as a function:

$$\boldsymbol{f(x) = 6x - 1}$$

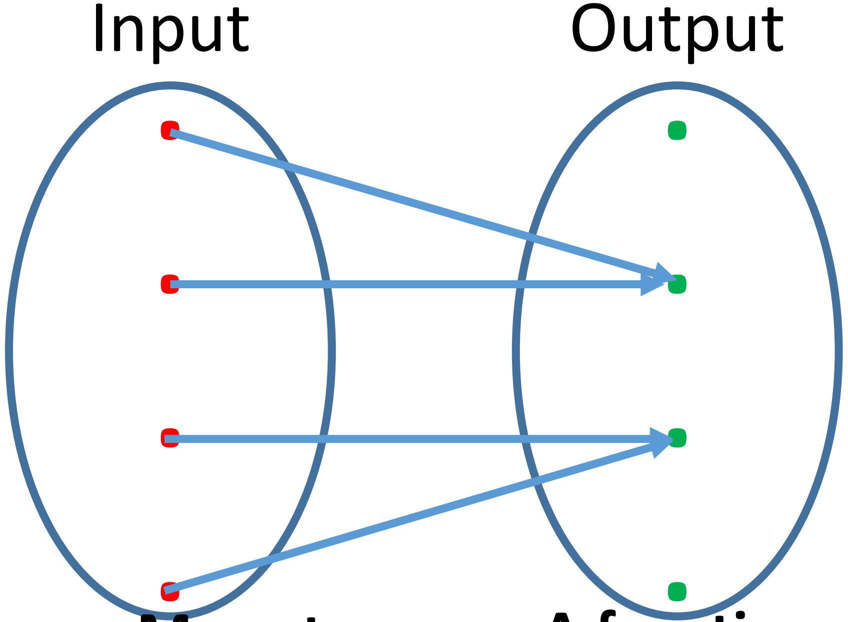
$$\boldsymbol{f(x)}$$



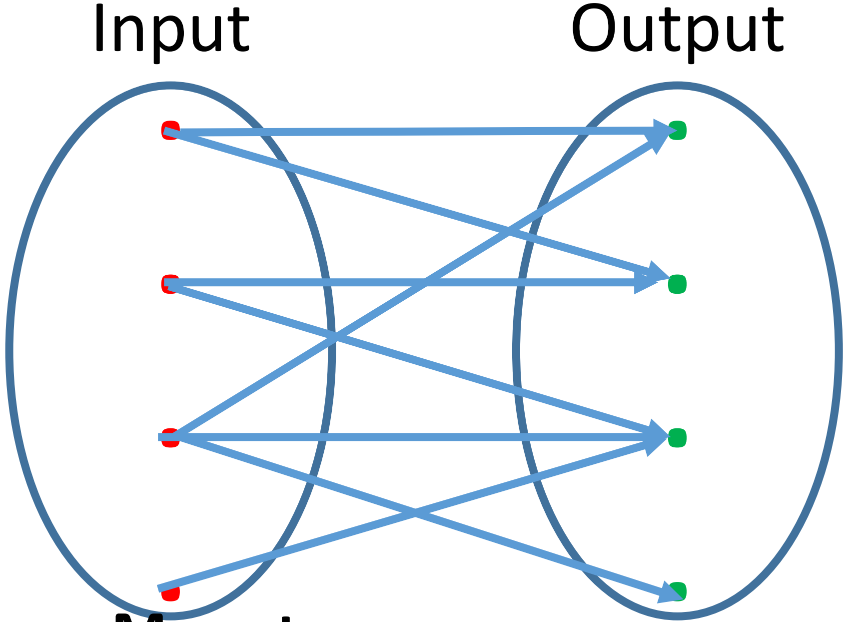
One to one
A function



One to many
Not a Function



Many to one **A function**

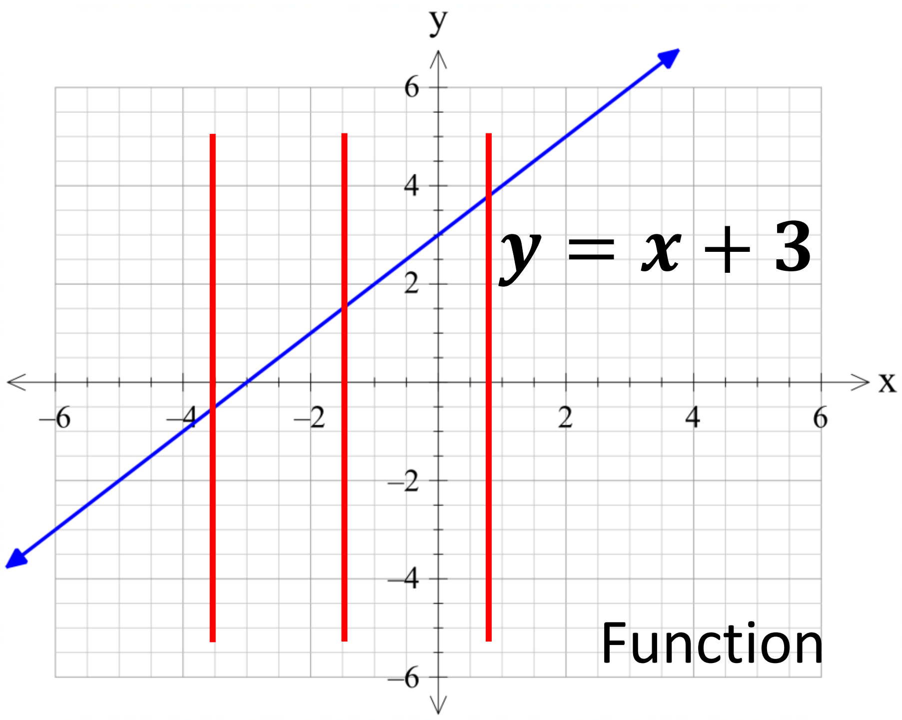


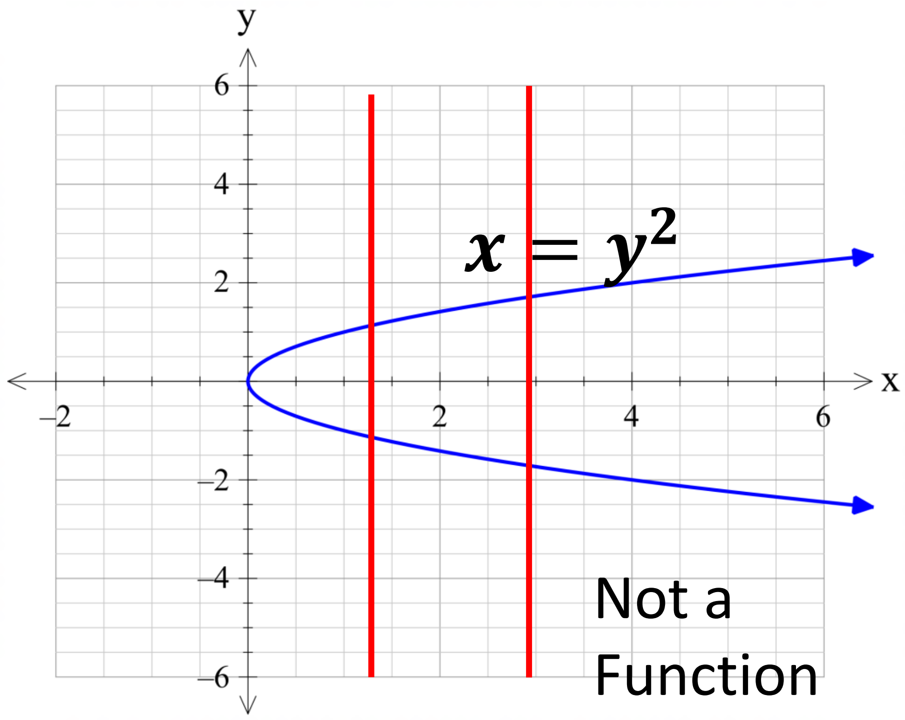
Many to many **Not a Function**

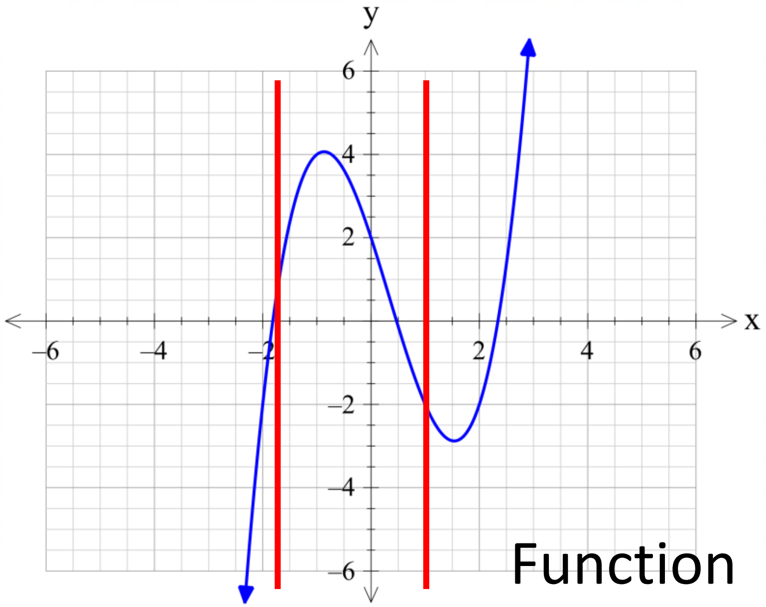
Vertical Line Test

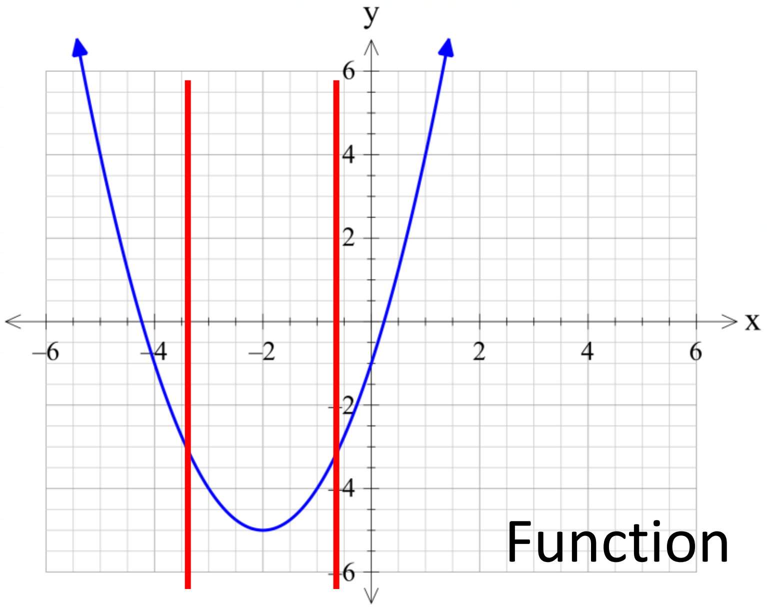
If we draw all possible vertical lines on the graph of a relation, the relation:

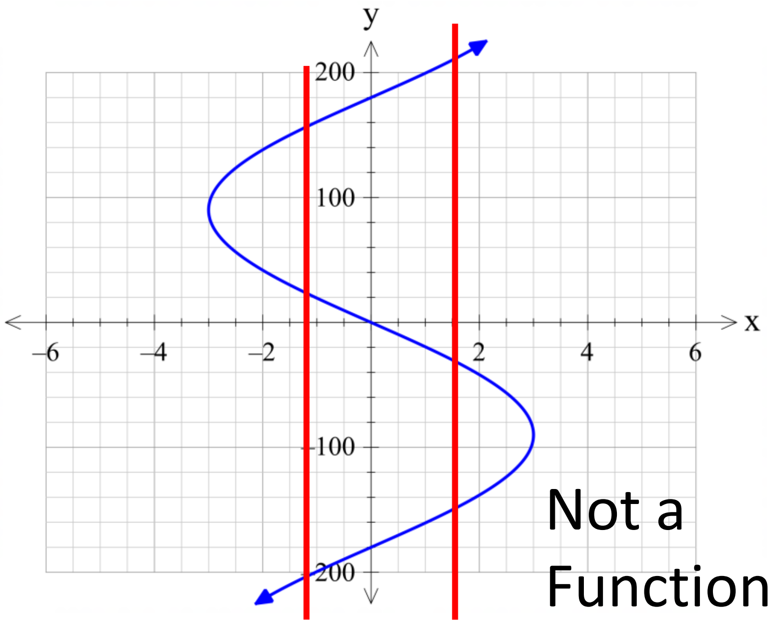
- is a **FUNCTION** if each line cuts the graph no more than once; and
- Is **NOT** a function if **AT LEAST** one line cuts the graph more than once.

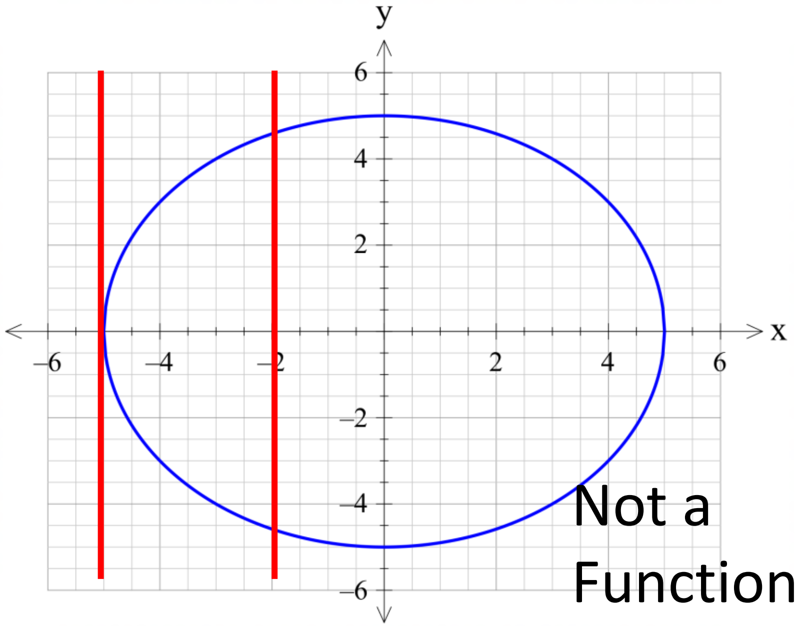




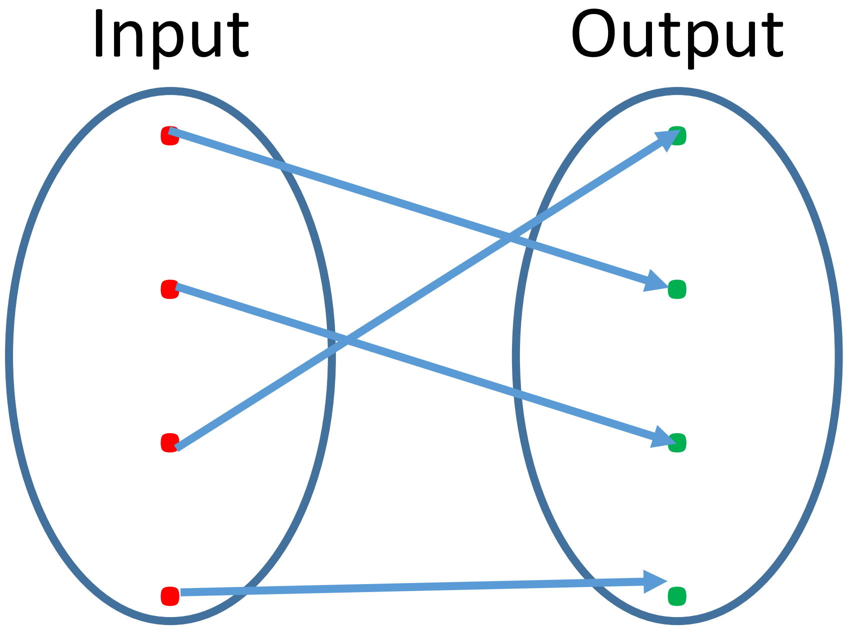

y
x
6
4
2
−2
−4
−6
−6
−4
−2
2
4
6
Function


y
x
Function

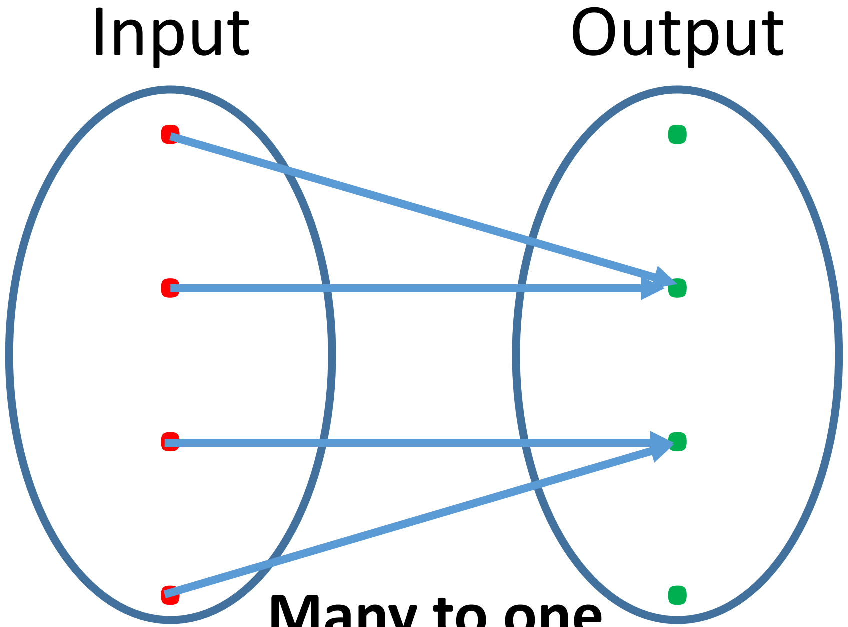

y
x
200
100
−100
−200
Not a Function


y
x
Not a Function

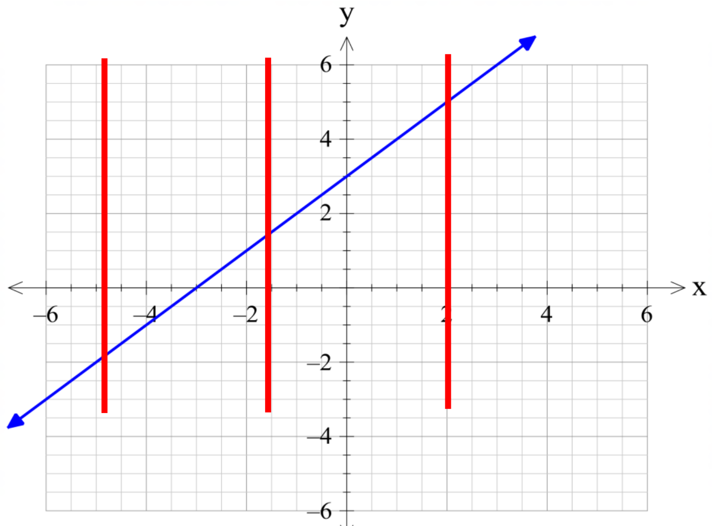
Functions

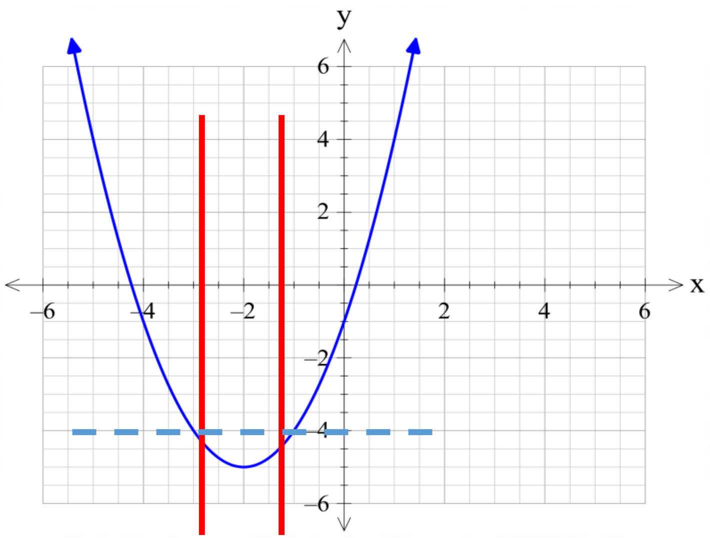


One to one

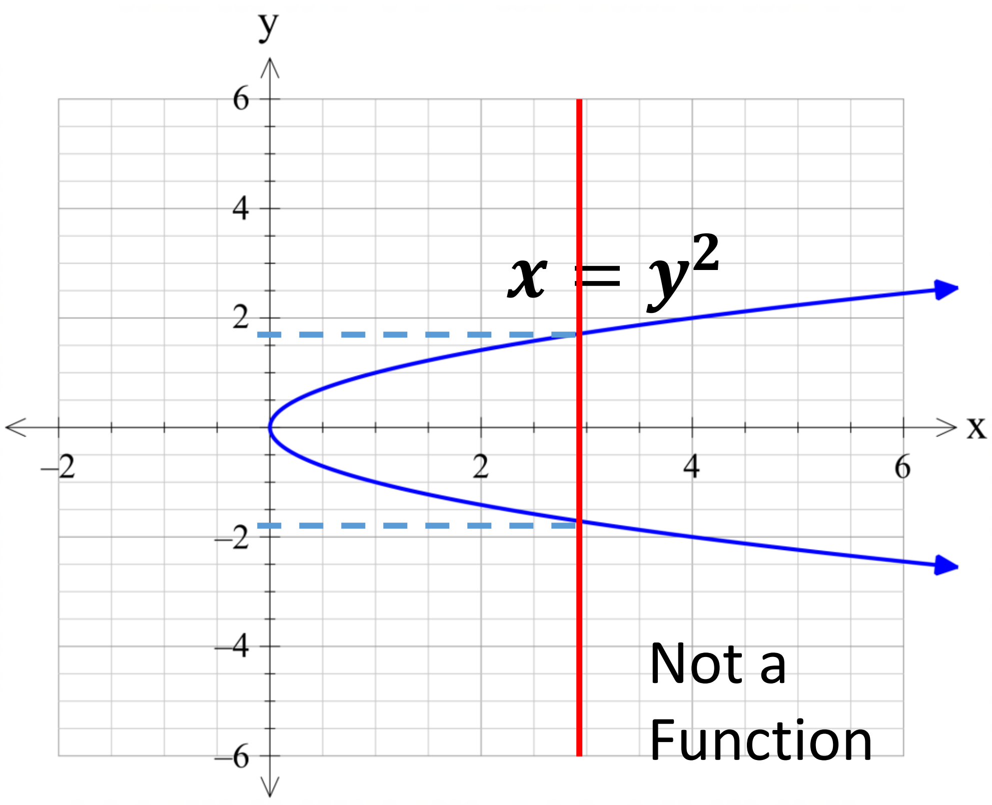


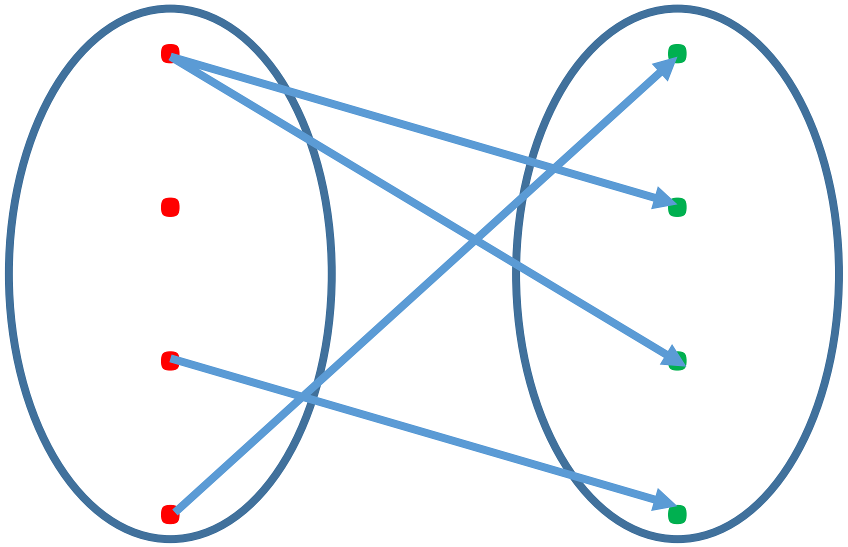
Many to one

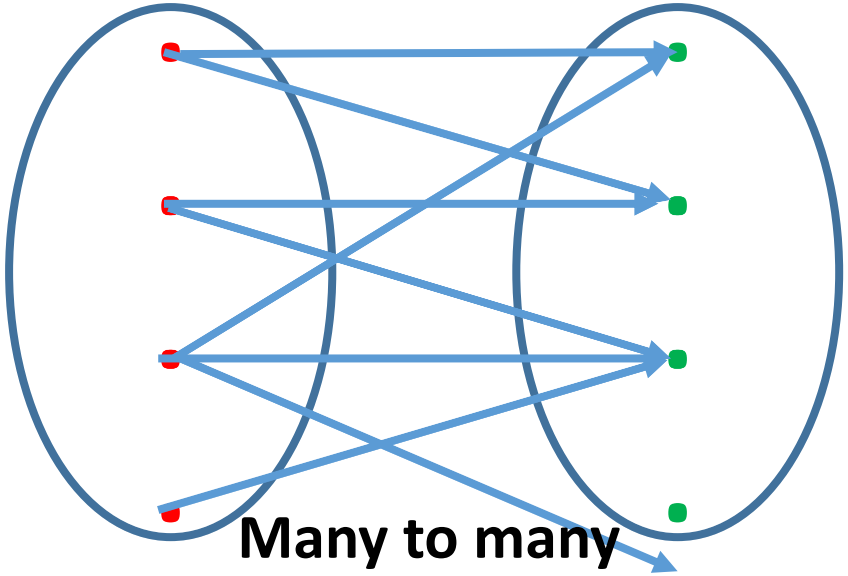




Not function







Guided Practice

Which of the following sets of ordered pairs defines a function?

a) $\{(-3,-4),(-1,-1),(-6,7),(1,5)\}$

b) $\{(-4,1),(-4,-1),(-6,7),(-6),8)\}$

a) $\{(-3,-4),(-1,-1),(-6,7),(1,5)\}$ is a function because for each x value, there is only one y value

b) $\{(-4,1),(-4,-1),(-6,7),(-6),8)\}$ is not a function because there is an x value, with 2 different y value

Domain and Codomain

A function f consists of

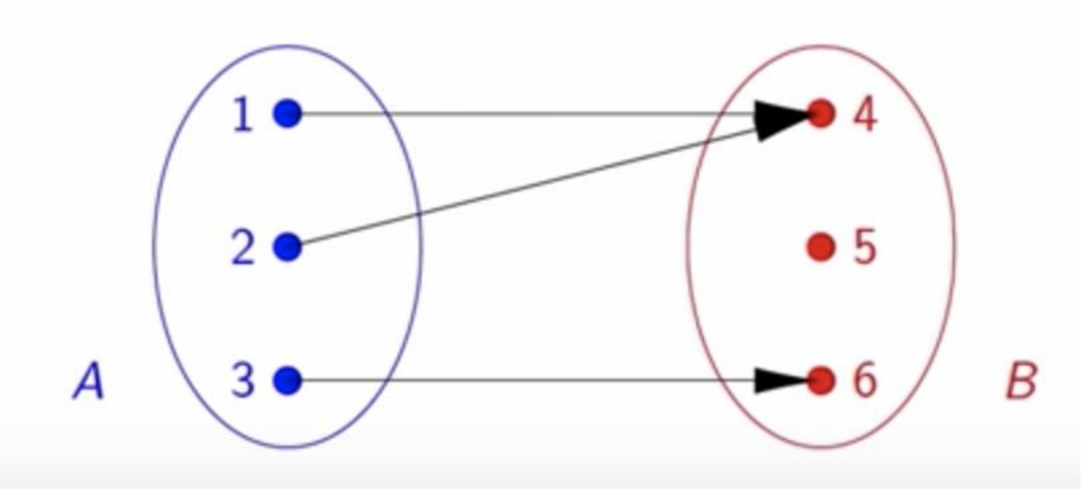
- a **DOMAIN**: set of inputs
- a **CODOMAIN**: set of "potential" outputs
- a relation (or rule) which matches each input to exactly one output

EXAMPLE

{1, 2, 3}

{4, 5, 6}

$f(1) = 4$
$f(2) = 4$
$f(3) = 6$



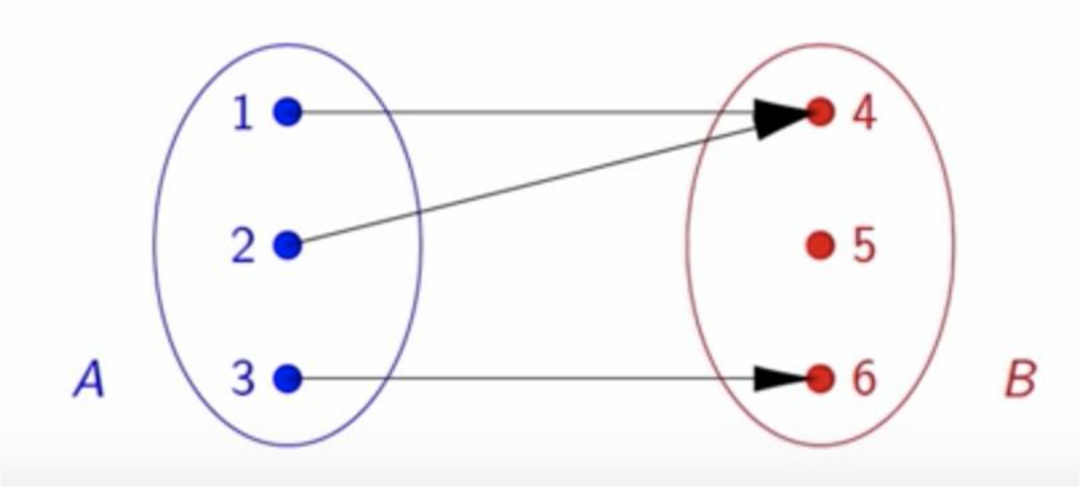
Codomain and Range

- a **CODOMAIN**: set of “potential” outputs
- a **RANGE**: set of “actual” outputs

EXAMPLE

{4, 5, 6}

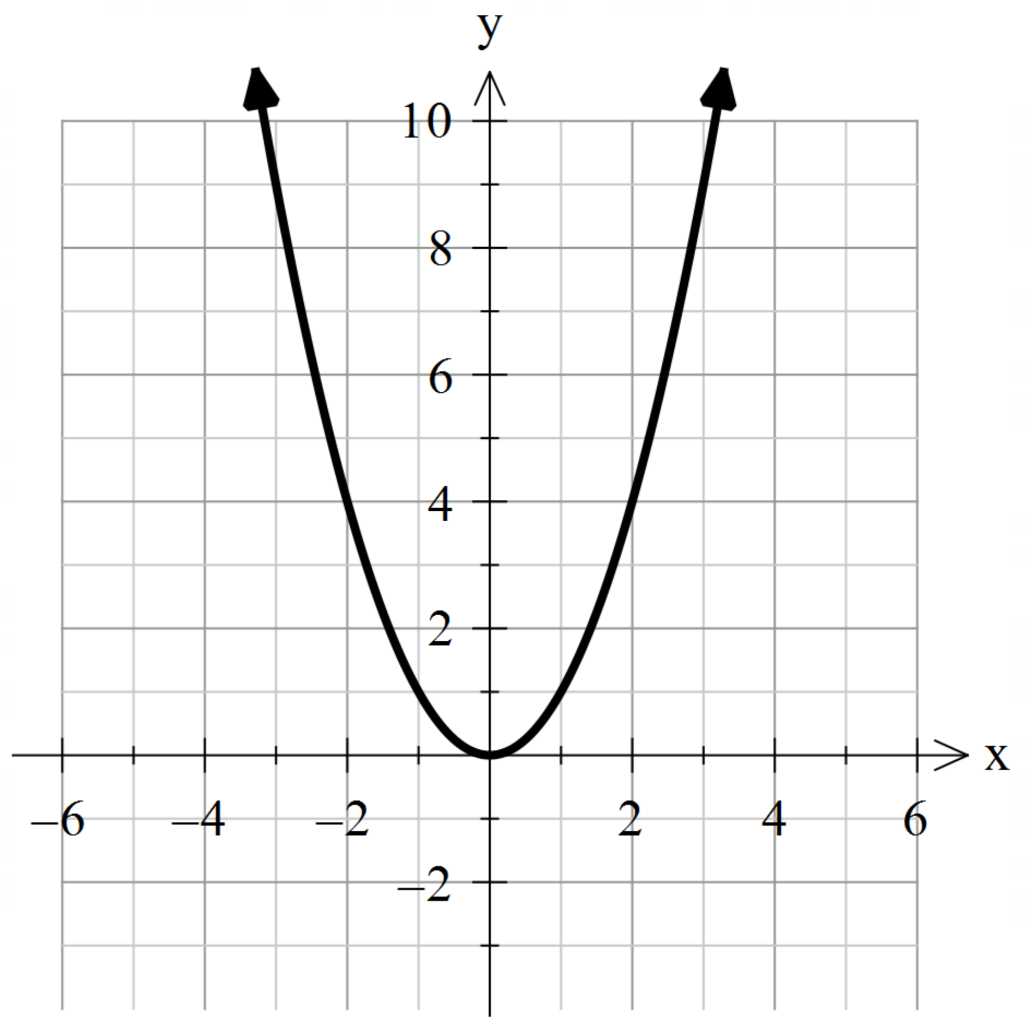
{4, 6}



We need to specify the **DOMAIN** of a function but DO NOT need to specify the **RANGE** as we can calculate the range.

Guided Practice

a) Is $y = x^2$ a function? State the maximal domain and range.
b) Is $x^2 + y^2 = 4$ a function? State the maximal domain and range.



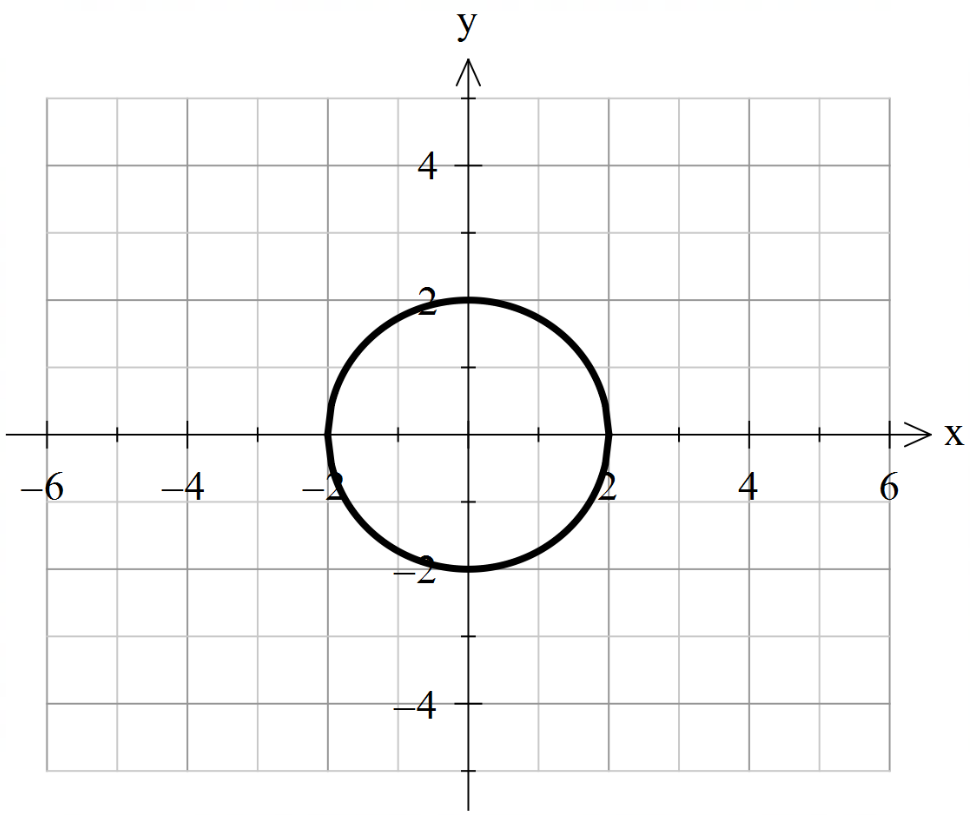
a) $y = x^2$ is a function
From the vertical line test, for each x-value, there is only one y-value

Domain: $\{x: x \in \mathbb{R}\}$

Range: $\{y: y \geq 0\}$

Guided Practice

a) Is $y = x^2$ a function? State the maximal domain and range.
b) Is $x^2 + y^2 = 4$ a function? State the maximal domain and range.



b) $x^2 + y^2 = 4$ is not a function
From the vertical line test, for each x-value, there are more than one y-values

Domain: $\{x: -2 \leq \mathrm{x} \leq 2\}$

Range: $\{\mathrm{y}: -2 \leq \mathrm{y} \leq 2\}$

Guided Practice

Domain ↙ ↙ Co-domain

Rewrite the following using $f: X \to Y$ notation

$$\{(x, y): y = -3x + 2\}$$

Determine the Domain $= \mathbb{R}$

Determine the Codomain $= \mathbb{R}$

Determine the function $f(x) = -3x + 2$

$$f: \mathbb{R} \to \mathbb{R}, \qquad f(x) = -3x + 2$$

Guided Practice

Rewrite the following using $f: X \to Y$ notation

$$\{(x, y): y = 2x + 3, x \geq 0\}$$

Determine the Domain $= \mathbb{R}^+ \cup \{0\}$

Determine the Codomain $= \mathbb{R}$

Determine the function $f(x) = 2x + 3$

$$f: [0, \infty) \to \mathbb{R}, \qquad f(x) = 2x + 3$$

Guided Practice

Rewrite the following using $f: X \to Y$ notation

$$y = \frac{1}{x-2}, x \neq 2$$

Determine the Domain $= \mathbb{R} \setminus \{2\}$

Determine the Codomain $= \mathbb{R}$

Determine the function $f(x) = \frac{1}{x-2}$

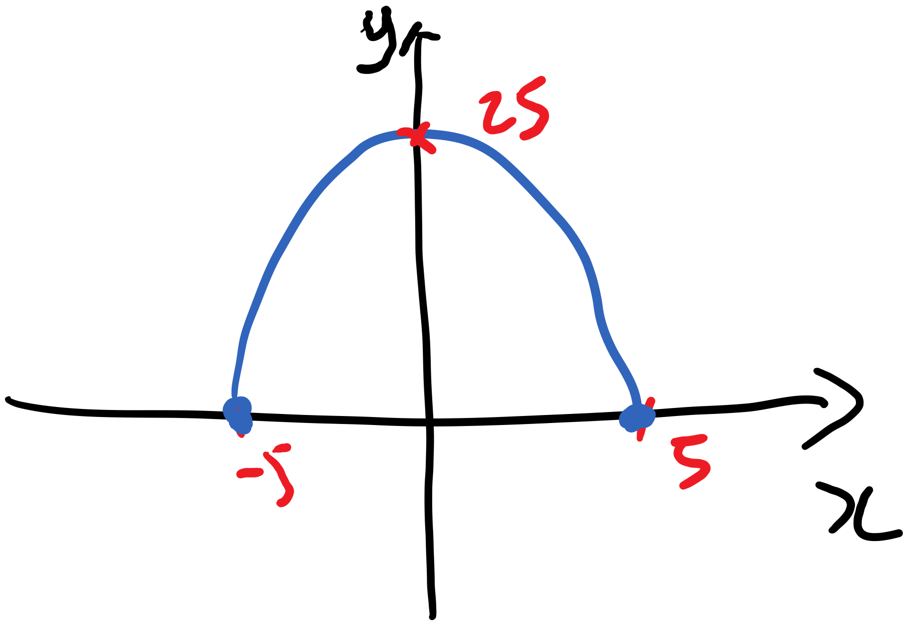
$$f: \mathbb{R} \setminus \{2\} \to \mathbb{R}, \qquad f(x) = \frac{1}{x-2}$$

Guided Practice

Rewrite the following using $f: X \to Y$ notation

$$y + x^2 = 25, -5 \leq x \leq 5$$

$$y = -x^2 + 25$$



Determine the Domain $= [-5,5]$

Determine the Codomain $= \mathbb{R}$

Determine the function $f(x) = -x^2 + 25$

$$f: [-5,5] \to \mathbb{R}, \qquad f(x) = -x^2 + 25$$

Guided Practice

Given $f(x) = 2x - 3$, evaluate:

a) $f(3)$ b) $f(2) - f(-1)$ c) $f(3) \times f(1)$ d) $f(p)$

$f(3) = 2(3) - 3$

$= 3$

$f(2) - f(-1)$

$= 2(2) - 3 - [(2(-1) - 3]$

$= 1 - (-5)$

$= 6$

$f(3) \times f(1)$

$= (2(3) - 3) \times (2(1) - 3)$

$= (3) \times (-1)$

$= -3$

$f(p) = 2p - 3$

Guided Practice

Given $f(x) = 2x - 4$,

a) Find the value of x for which $\mathrm{f}(x) = 6$

b) Find the value of x for which $\mathrm{f}(x) = 0$

c) Find the value of t for which $\mathrm{f}(t) = t$

a) $2x - 4 = 6$

$2x = 10$

$x = 5$

b) $2x - 4 = 0$

$2x = 4$

$x = 2$

c) $2t - 4 = t$

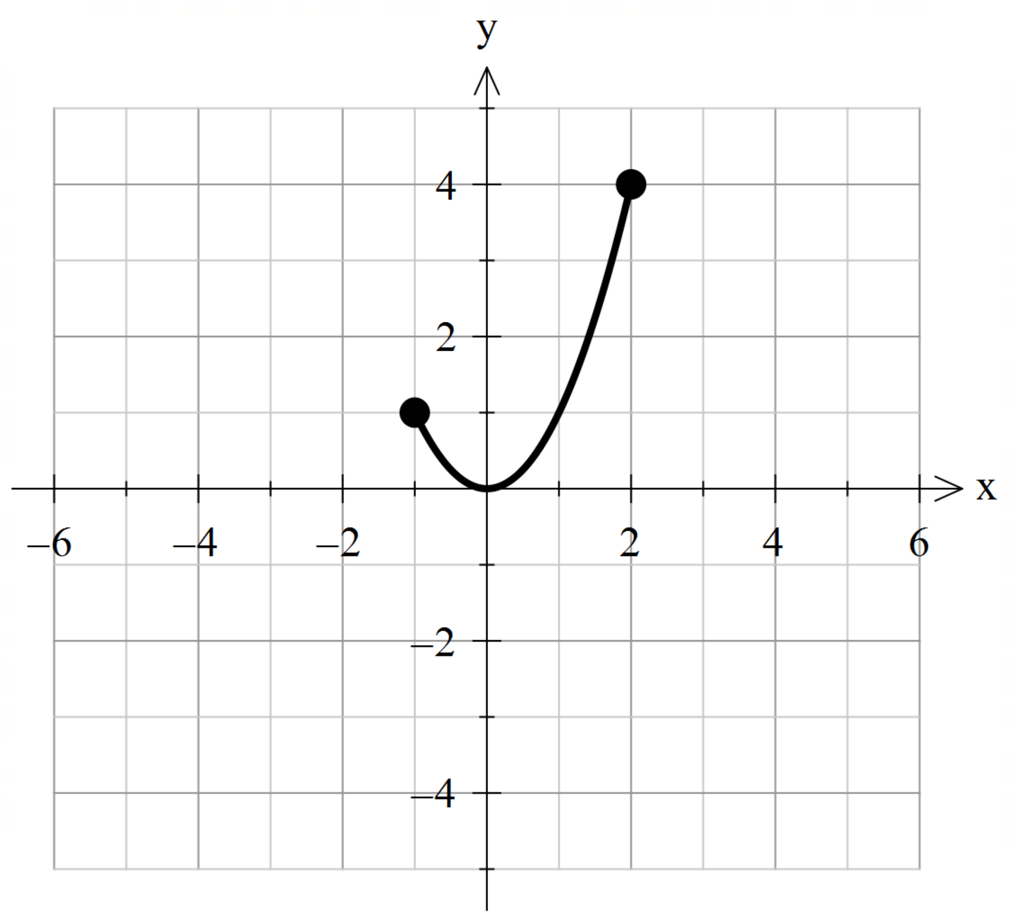
$t = 4$

Guided Practice

Sketch the graph and state the range

$$f : [-1,2] \to \mathbb{R}, \qquad f(x) = x^2$$

Range : $[0, 4]$



Guided Practice

Sketch the graph and state the range

$$f : [-2,2] \to \mathbb{R}, \qquad f(x) = x^2 + 2x$$

Factorise to find x-intercept:

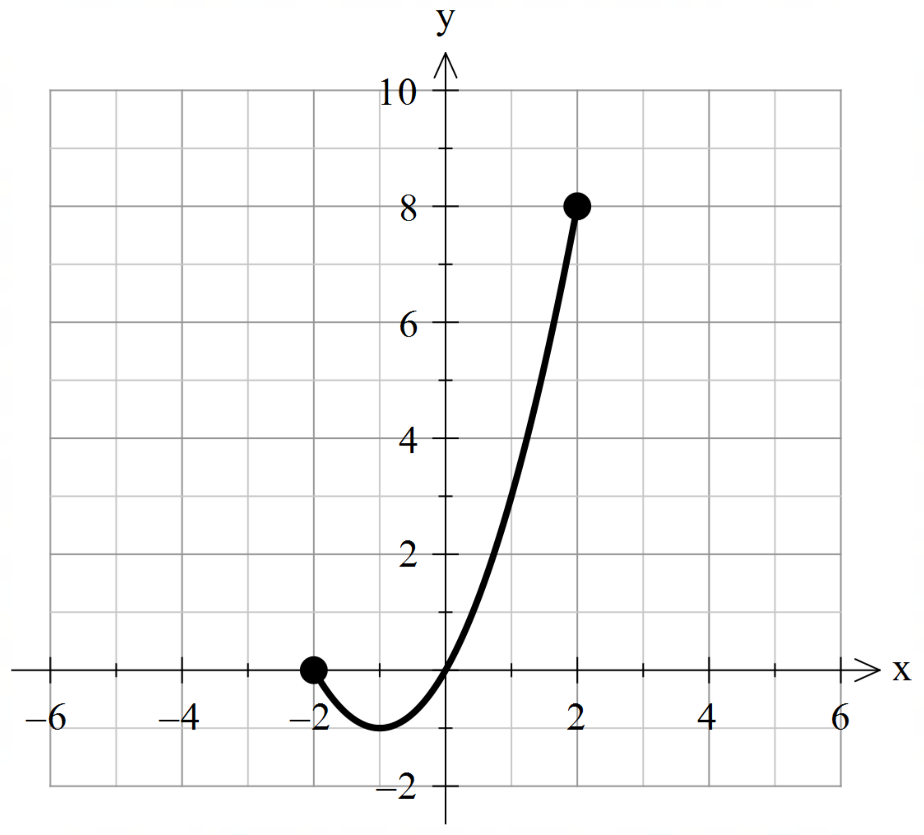
$f(x) = x(x+2)$

$(0,0), (-2,0)$

Find turning point:

$(-1,-1)$

Range : $[-1, 8]$

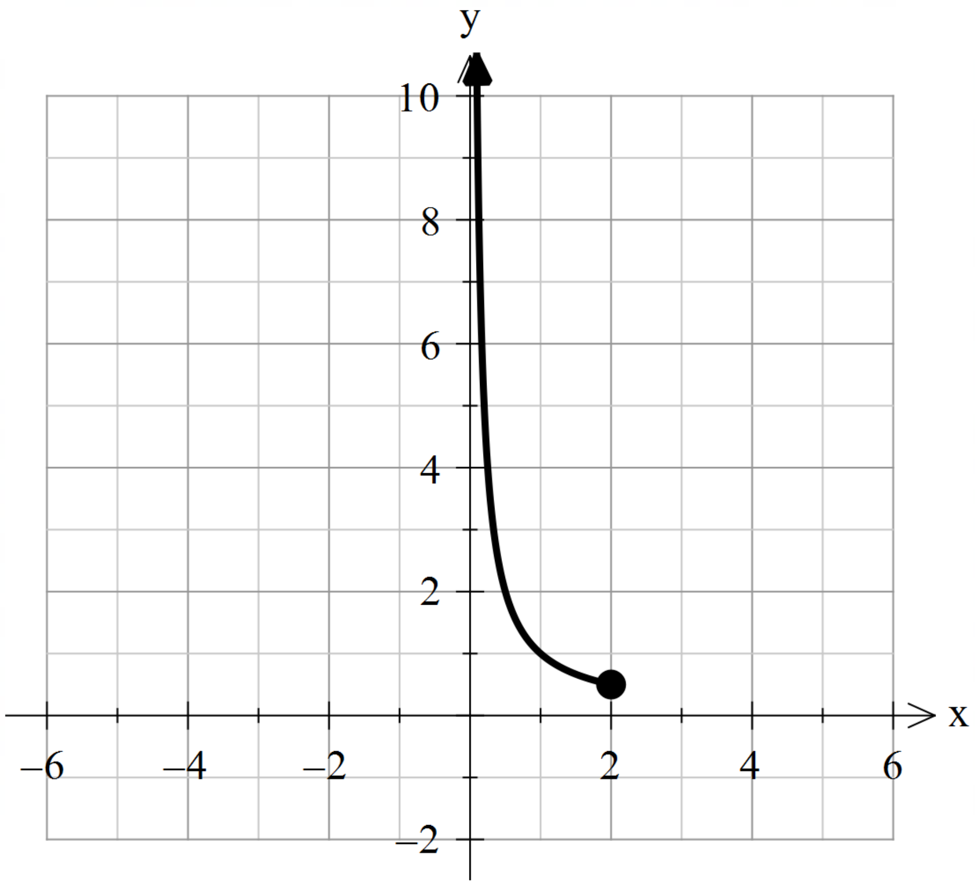


Guided Practice

Sketch the graph and state the range

$$f: (0,2] \to \mathbb{R}, \qquad f(x) = \frac{1}{x}$$

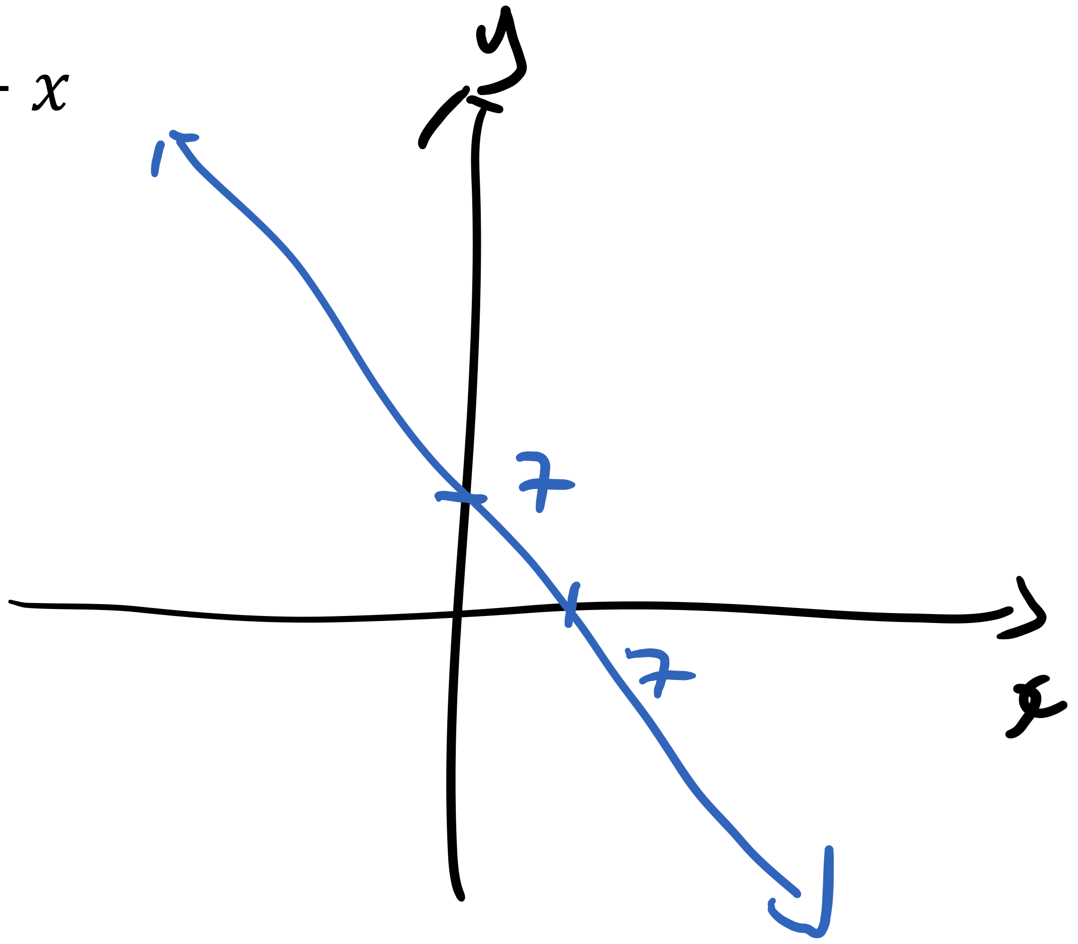
Range : $\left[\frac{1}{2}, \infty\right)$



Guided Practice

Find the domain and range of the following functions

$$f(x) = 7 - x$$



Domain $= \mathbb{R}$

Range $= \mathbb{R}$

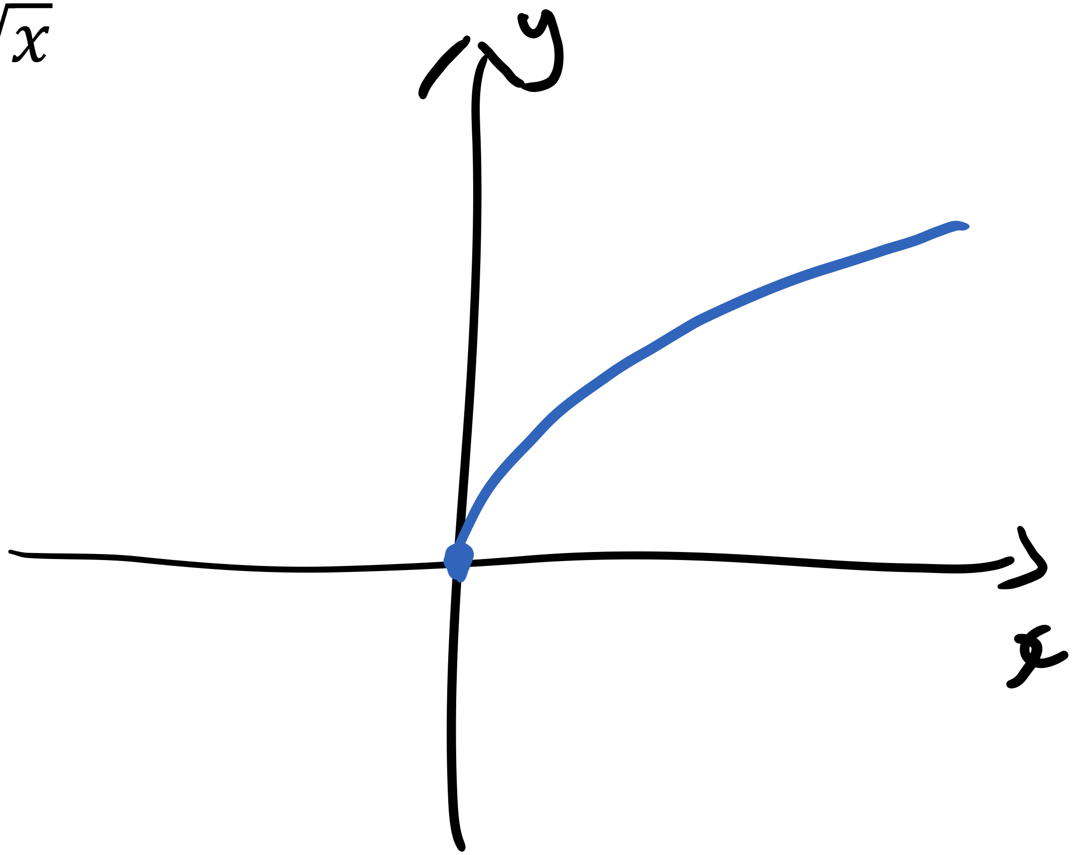
Guided Practice

Find the domain and range of the following functions

$$f(x) = 2\sqrt{x}$$

Domain $= [0, \infty)$ or $\mathbb{R}^{+} \cup \{0\}$

Range $= [0, \infty)$ or $\mathbb{R}^{+} \cup \{0\}$



Guided Practice

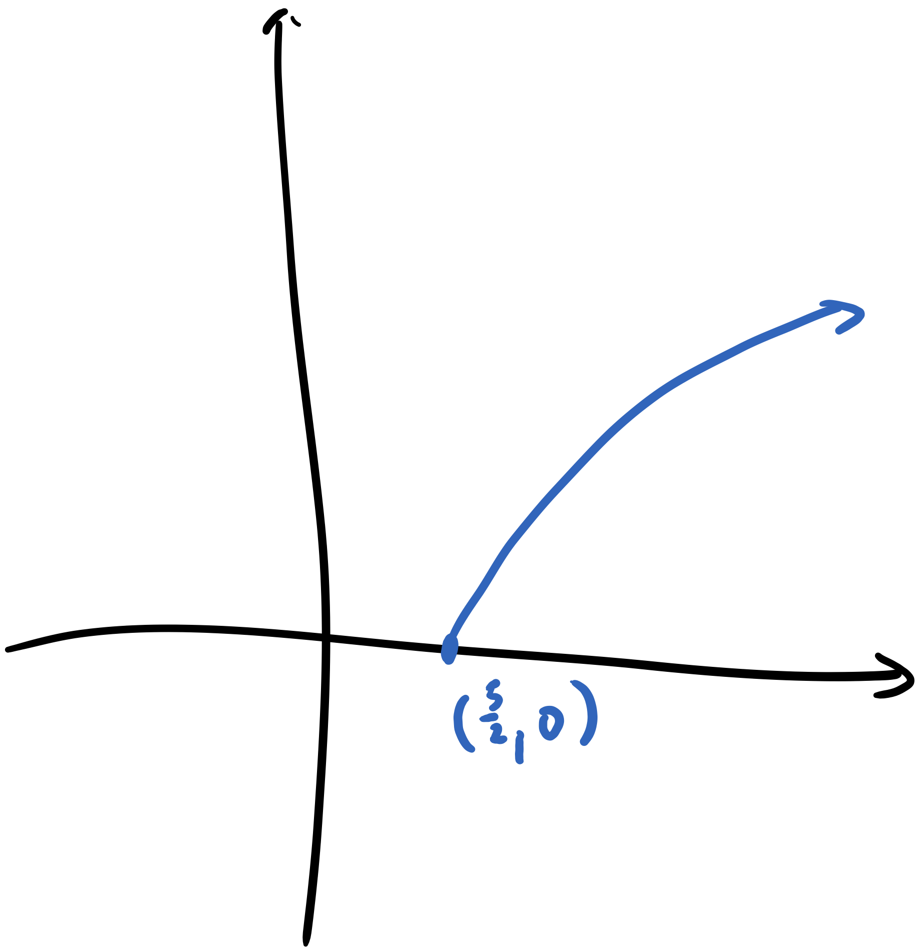
Find the domain and range of the following functions

$$f(x) = \sqrt{2x - 5}$$

$$f(x) = \sqrt{2\left(x - \frac{5}{2}\right)}$$

Domain $= \left[\frac{5}{2}, \infty\right) = \left\{x : x \geq \frac{5}{2}\right\}$

Range $= [0, \infty) = \{y : y \geq 0\}$



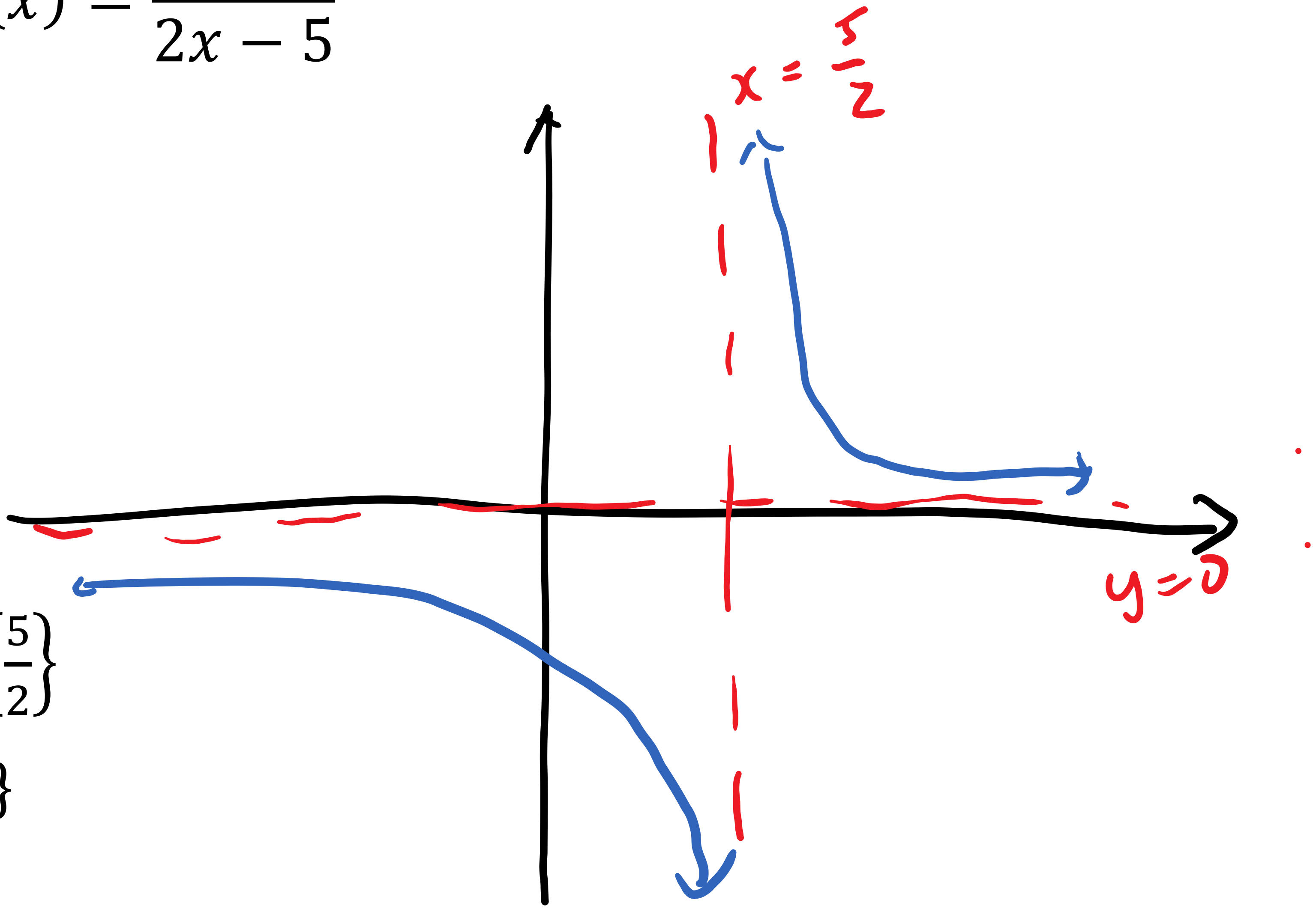
Guided Practice

Find the domain and range of the following functions

$$f(x) = \frac{1}{2x - 5}$$

$$f(x) = \frac{1}{2\left(x - \frac{5}{2}\right)}$$

$$x \neq \frac{5}{2}, \qquad y \neq 0$$



Domain $= \left\{x : x \neq \frac{5}{2}\right\} = \mathbb{R} \setminus \left\{\frac{5}{2}\right\}$

Range$= \{y : y \neq 0\} = \mathbb{R} \setminus \{0\}$

Guided Practice

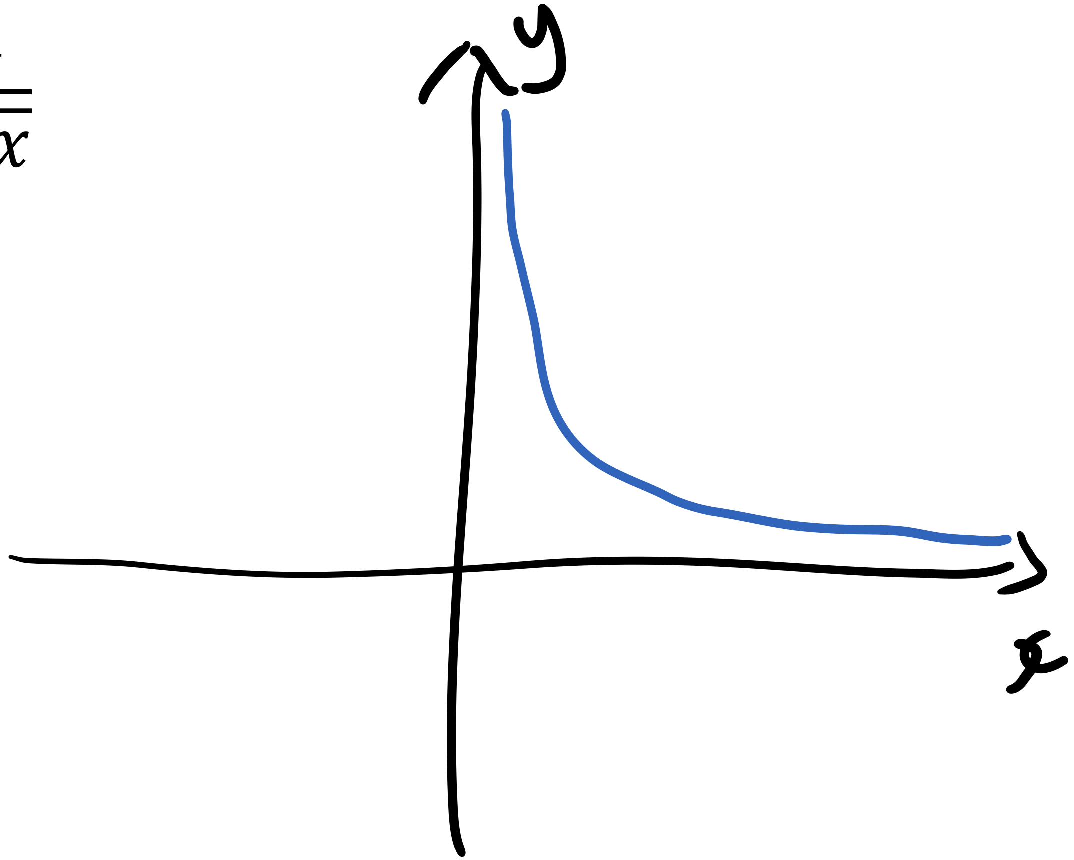
Find the domain and range of the following functions

$$f(x) = \frac{1}{\sqrt{x}}$$

$$x > 0$$

Domain $= (0, \infty)\ or\ \mathbb{R}^+$

Range $= (0, \infty)\ or\ \mathbb{R}^+$



Independent Practice

Complete Cambridge Ex 6C